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SQUARE DIVISOR CORDIAL LABELING FOR SOME GRAPHS

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ABSTRACT
Let G = {V (G), E (G)} be a simple graph and f: v→ {1, 2. . . |v|} be a bijection. For each edge uv, assign the label 1. If either $[f(u)]^2 / f(v)$ or $[f(v)]^2 / f(u)$ and the label 0 otherwise f is called a square divisor cordial labeling if $|e_f\ (0) - e_f\ (1)| \leq 1$. A graph with a square divisor cordial labeling is called a square divisor cordial graph. In this work, a discussion is made on Shell, Tensor product, Coconut tree, Jelly fish and Subdivision of bistar under square divisor cordial labeling.

KEYWORDS: SDCL, Tensor Product, Coconut Tree, Jelly Fish, Shell graph.

INTRODUCTION
A graph $G(V,E)$ is of vertices and edges. The vertex set $V\,(G)$ is non-empty set and the edge set $E(G)$ may be empty. A vertex is simply an element of the vertex set and an edge represents a connection between two elements of an unordered pair from the vertex set. Labeling of graphs subject to certain condition gave raise to enormous work which listed by J. A. Gallian [1]. Square Divisior Cordial Labeling were introduced by Murugesan [3]. Let $G = \{V\,(G), E(G)\}$ be a simple graph, For each edge, assign the label 1 . If either $[f(u)]^2 / f(v)$ or $[f(v)]^2 / f(u)$ and the label 0 otherwise f is called a square divisor cordial labeling if $|e_f\ (0)\ - e_f\ \ (1)| \ \leq\ 1$. S. K. Vaidya, N. A. Dani and K. K. Kanani have proved that shells are cordial and 3-equitable graphs [5] , S. K. Vaidya and Lekha Bijukumar derived the tensor product $K_{1,n}$ and P_2 on odd graceful labeling of some graphs [6] , R. Uma and D. Amuthavalli proved the coconut tree $CT(m,n)$ and jelly fish $J(m,m)$ have proved fibonacci graceful labeling for some star related graphs [4] and subdivision of the central edge of the bistar $B_{n,n}$ have proved K. Murugan and A. Subramanian in labeling of subdivided graphs [2] have been discussed in this paper.

DEFINITIONS
Tensor Product:
The Tensor product $G_1 \circledast G_2$ of two simple graphs G_1 and G_2 is the graph with $V(G_1 \circledast G_2) = V_1V_2$, Where (u_1,u_2) and (v_1,v_2) are adjacent in G_1, G_2 if and only if u_1 is adjacent to v_1 in G_1 and u_1 is adjacent to v_2 in G_2 . The Tensor product of $K_{1,n}$ and P_2 .

Coconut Tree:
A Coconut Tree $CT(m,n)$ is the graph obtained from the path P_m by appending n new pendent edges at an end vertex of P_m.

Jelly Fish:
The Jelly fish graph $J(m,n)$ is obtained by joining a 4-cycle whose vertices are v_1, v_2, v_3, v_4 with vertices v_1 and v_3 defined by an edge and appending m pendent edges to v_2 and n pendent edges to v_4.

Subdivision of a graph:
A G is a graph that can be obtained from G by a sequence of edge subdivisions are called Subdivision of a graph. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions.

Shell graph:
A shell S_n is the graph obtained by taking $n-3$ concurrent chords in a cycle C_n on n vertices. The vertex at which all the chords are concurrent is called the apex vertex. The shell is also called fan F_{n-1}. i.e. $S_n = F_{n-1} = P_{n-1} + K_1$.

RESULTS

Theorem:
The shell graph S_n is a Square Divisor Cordial Graph.

Proof:
Let $G = S_n$ be the shell graph. By the definition of shell graph the order and size of G are $p = n + 1$ and $q = 2n - 1$. Define the vertex set by V = {x, u1, u2, · · ·, un} , where x be an apex vertex, u_i 's are adjacent to the apex vertex and u_i 's are connected by successive vertices. Define the edge set E as $E = E_1 \cup E_2$ where,
$E_1 = e_i = (x, u_i)$ and $E_2 = e_{ii} = \{(u_1, u_2), (u_2, u_3), \ldots, (u_{n-1}, u_n)\}$, where $i \in N$. We define the labeling $f : V(S_n) \to \{1, 2, 3, \ldots, n + 1\}$, where
$f(x) = 1$;
$f(u_i) = i + 1, \forall 1 \leq i \leq n$;
Therefore, the Shell graph S_n is a Square Divisor Cordial graph.

Example:

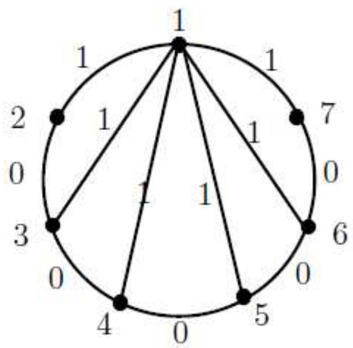

Fig 3.1 SDCL of Shell graph S_6

Square Divisor Cordial labeling of the Shell graph S_6 is shown in the Fig 3.1.
$|5 - 6| \leq 1$.
Hence, the given Shell graph S_6 is Square Divisor Cordial Graph.

Theorem
The Tensor Product graph (G1 (Tp) G2) is a Square Divisor Cordial Graph.

Proof:
Let $G = G_1(T_p)G_2$ be a tensor product graph of order $p = 2(n + 1)$ and size $= 2n$. Let the vertex $V = \{u, u_1, u_2, \ldots, u_n, v, v_1, v_2, \ldots, v_n\}$, where u, v are the apex vertices, u_i's are adjacent and pendent vertices of u and v_i 's are adjacent and pendent vertices of v . The edge set E is defined as $= \{e_1, e_2\}$, where $e_1 = e_i = (u, u_i)$ and $e_2 = e_{ii} = (v, v_i)\ \forall\, i = 1, 2, \ldots, n$. The bijective function $f : V(G) \to \{1, 2, 3, \cdots, 2n + 1\}$ is defined as,
$f(u) = 1$; $f(v) = 2$;
$f(u_i) = 2i + 2, \forall 1 \leq i \leq n$;
$f(v_i) = 2i + 1, \forall 1 \leq i \leq n$;
Such that $e_f(0) = n$ and $e_f(1) = n$.
Therefore, by the definition of Square Divisor Cordial Graph, $|e_f(0) - e_f(1)| \leq 1$.
$|n - n| \leq 1$.
Hence, the Tensor product graph $G_1(T_p)G_2$ is a Square Divisor Cordial Graph.

Example:
Square Divisor Cordial labeling of Tensor Product $(G_1(T_p)G_2)$ shown in Fig 3.2.

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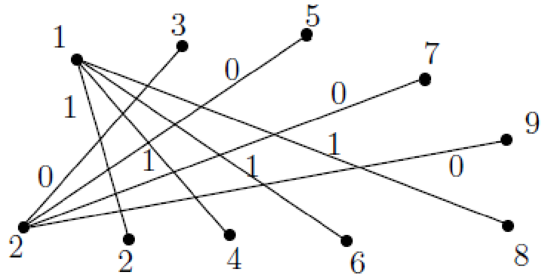

Fig 3.2 SDCL of Tensor Product

In this Fig 3.2, $e_f\ (0) = 4$ and $e_f\ (1) = 4$.
By the definition of Square Divisor Cordial Labeling,
$|e_f\ (0) - e_f\ (1)| \leq 1$,
$|4 - 4| \leq 1$.
Hence, the Tensor product graph $G_1(T_p)G_2$ has a Square Divisor Cordial Labeling.

Theorem:
The Coconut Tree $CT(n, n - 2)$ is a Square Divisor Cordial Graph.
Proof:
Let $CT(n, n - 2)$ be the Coconut Tree. The order of $CT(n, n - 2)$ is $p = n + n - 2$ and size $= n + n - 3$. By the definition of Coconut Tree, the vertex set $(G) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_{n-2}\}$. Let $u_1, u_2, \dots, u_n$ be the vertices of the path P_n and $v_1, v_2, \dots, v_{n-2}$ be the pendent vertices attached by the beginning vertex of path P_n. The edge set $E(G) = \{e_{ni}, e_{ij}\}$, where $e_{1i} = (u_1, v_i)$ and $e_{ij} = (u_i, u_j)\ \forall\ i, j = 1, 2, \dots, n$
Let us define the function $f : V(G) \to \{1, 2, \dots, n + n - 2\}$;
$f(u_i) = i, \forall 1 \leq i \leq n$;
$f(v_1) = q + 1$;
$f(v_{1+1}) = q + 1 - i, \forall 1 \leq i \leq n$;
Such that $e_f\ (0) = n - 2$ and $e_f\ (1) = n - 1$,
$|n - 2 - n + 1| \leq 1$;
$|e_f\ (0) - e_f\ (1)| \leq 1$.
Which is condition of the Square Divisor Cordial Labeling. Thus the Coconut Tree $CT(n, n - 2)$ is a Square Divisor Cordial Graph.

Example:
Square Divisor Cordial Labeling of $CT(5, 3)$ shown in Fig. 3.3.

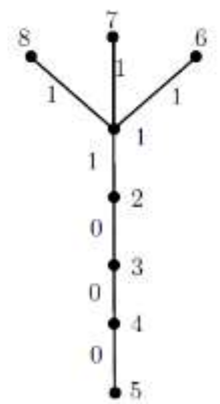

Fig 3.3 SDCL of $CT(5, 3)$

$|e_f\ (0) - e_f\ (1)| \leq 1 = |3 - 4| \leq 1$.
Hence, the Coconut Tree CT (5, 3) is a Square Divisor Cordial Labeling.

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Theorem:
The graph Jelly Fish J (m, m) is Square Divisor Cordial Graph.
Proof:
Let $G = J(m,m)$ of order $p = 2m + 4$ and size $q = 2m + 5$ be a jelly fish. The vertex set and edge set of $J(m,m)$ are
$V\,(J(m,m)) = \{[u,v,x,y],[u_i,v_i\,\forall\,1 \le i \le m]\}$
$E(J(m,m)) = \{[(ux) \cup (uy) \cup (vx) \cup (vy) \cup (xy)] \cup [(uu_i)\,\forall\,1 \le i \le m] \cup [(vv_i),\forall\,1 \le i \le m]\}$
Let us define the function $f : V\,(G) \to \{1,2,\dots,2m + 4\}$ as follows:
$f(u) = 1\,;\, f(v) = 2\,;$
$f(x) = 2m + 3\,;$
$f(y) = 2m + 4\,;$
$f(u_i) = 2i + 2\,,\forall\,1 \le i \le n\,;$
$f(v_i) = 2i + 1\,,\forall\,1 \le i \le n$. The above function induces $e_f\ (0) = m + 2$ and $e_f\ (1) = m + 3$
$|m + 2 - (m + 3)| \le 1.$
Hence, the graph Jelly Fish $J(m,m)$ is a Square Divisor Cordial Graph.

Example:
The Jelly Fish $J(4,4)$ is shown in Fig 3.4.
$e_f\ (0) = 6$ and $e_f\ (1) = 7$,
$\left|e_f\ (0) - e_f\ (1)\right| \le 1$,
$|6 - 7| \le 1$.

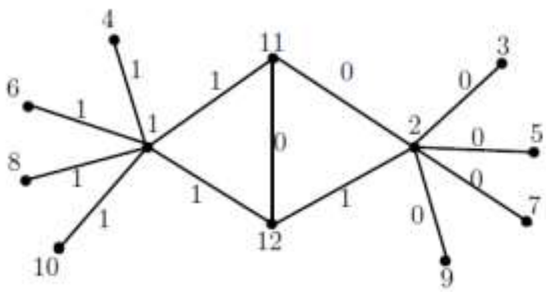

Fig 3.4 SDCL of $\boldsymbol{J(4,4)}$

Hence, the graph Jelly Fish $J(4,4)$ is a Square Divisor Cordial Labeling.

Theorem:
The graph obtained by the Subdivision of the central edge of the bistar $(B_{n,n})$ has a Square Divisor Cordial Graph.
Proof:
Let G be the graph obtained by the Subdivision of the central edge of the bistar $B_{n,n}$. Let the vertex set and edge set of the graph $B_{n,n}$ are
$V\,(G) = \{u,v,w,u_i,v_i\}\,,\forall\,1 \le i \le n\,;$
E(G) = {uw,wv, uu_i, vv_i} , ∀ 1 ≤ i ≤ n ;
The order of G is $p = 2n + 3$ and the size $= 2n + 2$. Let us define the function $f : V\,(G) \to \{1,2,\cdots,2n + 3\}$ by
$f(u) = 1\,;\, f(v) = 2\,;\, f(w) = 3\,;$
$f(u_i) = 2i + 2\,,\forall\,1 \le i \le n\,;$
$f(v_i) = 2i + 3\,,\forall\,1 \le i \le n\,;$
$e_f\ (0) = n + 1$ and $e_f(1)$ = n + 1.
Therefore by the definition of Square Divisor Cordial Graph, $|e_f\ (0) - e_f\ (1)| \le 1$.
Hence, the Subdivision of the central edge of the bistar $(B_{n,n})$ is a Square Divisor Cordial Graph.

Example:
The Square Divisor Cordial Labeling of Subdivision $B_{4,4}$ is shown in Fig 3.5.

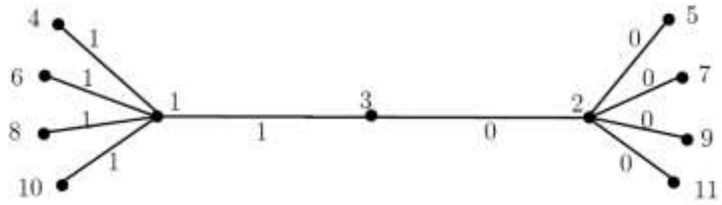

Fig 3.5 SDCL of $B_{(4,4)}$

$e_f(0) = 5$ and $e_f(1) = 5$,
$|e_f(0) - e_f(1)| \leq 1$.
Hence, the Subdivision of graph $B_{4,4}$ is a Square Divisor Cordial Labeling.

CONCLUSION
The overview of Square Divisor Cordial Graphs is the current interest due to its diversified applications. Here we investigate some results corresponding to labeled graphs. Similar work can be carried out for the other graphs also. The complied information related to SDCL will be useful for researchers to get some idea related to their field.

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